Buckling of Orthotropic, Curved, Sandwich Panels Subjected to Edge Shear Loads

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A linear analysis is presented for determining the shear buckling load of a rectangular, sandwich panel with constant cylindrical curvature and having specially orthotropic facings and core. Solution for classical simple-support boundary conditions was carried out by the Galerkin method, taking precautions not to "escalate" the equations by differentiation. This resulted in two doubly infinite sets of eigenvalue equations, representing buckling in the symmetric and unsymmetric modes. Numerical solutions were obtained on a digital computer and the results were compared with those of previous analyses for special cases and with limited test data available for the general case. Finally parametric design curves are presented for a range of parameters.

Nomenclature

$A_{mn}{}^i$ = integration coefficients defined in Appendix B
a = axial length of panel
a_{mn},b_{mn},c_{mn} = Fourier coefficients of $w(x,y)$, $Q_x(x,y)$, and $Q_y(x,y)$
b = circumferential width of panel
c = core depth
D_x,D_y = flexural rigidities of panel in the x and y directions
D_{xy} = twisting rigidity of panel
D_{qx},D_{qy} = thickness-shear rigidities of panel in the xz and yz planes
E_x,E_y = axial and circumferential Young's moduli of facings
E_s = Young's modulus of isotropic facings
E = Young's modulus of thin, homogeneous, isotropic panel
G_{xy} = in-plane shear modulus of orthotropic facings
G_{xz},G_{yz} = thickness shear moduli in the xz and yz planes
G_z = thickness shear modulus of isotropic core
H_1,H_2,H_3 = left-hand sides of Eqs. (1, 2, and 3)
i,j = dummy indexes
L_D,L_E = differential operators defined in Eqs. (4) and (5)
$L_E{}^{-1}$ = inverse differential operator defined by Eq. (6)
m = axial wave number (≡ number of axial half waves)
n = circumferential wave number (≡ number of circumferential waves)
N_{xy} = in-surface shear stress resultant, lb/in.
$(N_{xy})_{cr}$ = critical value of N_{xy} at which buckling is predicted
Q_x,Q_y = thickness-shear stress resultants acting in the xz and yz planes
R = radius of panel at middle surface
S_x,S_y,S_{xy} = extensional rigidities of panel
t = facing thickness
t_0 = thickness of ordinary (nonsandwich) panel
u,v,w = axial, circumferential, and normal deflections
x,y,z = orthogonal curvilinear coordinates on the panel middle surface in the axial, circumferential, and normal directions
δ_{mnij} = special delta function defined in Eq. (B12)
λ = eigenvalue defined in Eq. (19)
ν_{xy},ν_{yx} = Poisson's ratios corresponding to loading in the respective x and y directions
ν_s = Poisson's ratio of isotropic facings
ν = Poisson's ratio of thin, homogeneous, isotropic panel

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Introduction

THE buckling behavior of rectangular, curved, sandwich panels under compressive loadings has been described extensively in the literature.[1-4] However, the use of these panels as either wing coverings or as fuselage elements requires that the panels carry edgewise shear. This paper presents a theoretical analysis of the buckling behavior of rectangular, curved, orthotropic, sandwich panels under edgewise shear loading with simply supported boundaries. Examples of orthotropic facings are those of plywood or of filamentary composite materials (such as glass/epoxy, boron/epoxy, etc.). Orthotropic cores are exemplified by hexagonal-cell honeycomb core and corrugated core. Sandwich panels with isotropic facings (such as most structural alloys) and/or isotropic cores (such as foam core) and ordinary (thin, homogeneous, isotropic) panels are merely special cases of the general analysis presented here.

In the analysis, the Galerkin method is used, with special care taken not to escalate the governing differential equations.[5] This results in two eigenvalue equations describing buckling into either an even or an odd number of waves with the buckling load being the minimum predicted load. The results of this analysis compare well with earlier analyses for special cases. This analysis also is compared with limited test data.

Linear Buckling Analysis

For an orthotropic sandwich panel with uniform cylindrical curvature and subjected to edgewise shear loading N_{xy}, the equilibrium relations are based on Ref. 6, but a more modern notation is used here[7]

$$L_D(w) + (S_{xy}/R^2)L_E{}^{-1}(w_{,xxxx}) + 2N_{xy}w_{,xy} - D_{qx}{}^{-1}[D_xQ_{x,xxx} + (\nu_{xy}D_y + D_{xy})Q_{x,xyy}] - D_{qy}{}^{-1}[D_yQ_{y,yyy} + (\nu_{yx}D_x + D_{xy})Q_{y,xxy}] = 0 \quad (1)$$

$$Q_x + D_x(w_{,xxx} - D_{qx}{}^{-1}Q_{x,xx} + \nu_{yx}w_{,xyy} - \nu_{yx}D_{qy}{}^{-1}Q_{y,xy}) + (D_{xy}/2)(2w_{,xyy} - D_{qx}{}^{-1}Q_{x,yy} - D_{qy}{}^{-1}Q_{y,xy}) = 0 \quad (2)$$

$$Q_y + D_y(w_{,yyy} - D_{qy}{}^{-1}Q_{y,yy} + \nu_{xy}w_{,xxy} - \nu_{xy}D_{qx}{}^{-1}Q_{x,xy}) + (D_{xy}/2)(2w_{,xxy} - D_{qx}{}^{-1}Q_{x,xy} - D_{qy}{}^{-1}Q_{y,xx}) = 0 \quad (3)$$

where a subscript comma denotes differentiation with respect to the subscript variable following the comma. The linear differential operators L_D and L_E and the inverse operator L_E^{-1} are defined as follows:

$$L_D(\;) \equiv D_x(\;)_{,xxxx} + (\nu_{yx}D_x + 2D_{xy} + \nu_{xy}D_y)(\;)_{,xxyy} + D_y(\;)_{,yyyy} \quad (4)$$

$$L_E(\;) \equiv (S_{xy}/S_y)(\;)_{,xxxx} + [1 - (\nu_{xy}S_{xy}/S_x) - (\nu_{yx}S_{xy}/S_y)](\;)_{,xxyy} + (S_{xy}/S_x)(\;)_{,yyyy} \quad (5)$$

$$L_E^{-1}(w_{,xxxx}) = w_{,xxxx}/L_E(w_{,xxxx}) \quad (6)$$

The stiffnesses appearing in Eqs. (1-5) are related to the elastic properties and cross-sectional dimensions for five classes of plates in Appendix A.

To apply the Galerkin method correctly, it is necessary to obtain solution functions which satisfy both the force and deflection types of boundary conditions. For a cylindrically curved rectangular panel with respective axial and circumferential dimensions a and b, it was shown by Batdorf[8] that classical, simply supported boundary conditions are satisfied by the following deflection-function series:

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\sin(m\pi x/a)\sin(n\pi y/b) \quad (7)$$

If the buckling loads are to be found using the normal deflection function in Eq. (7), certain boundary conditions are implied for the axial and circumferential deflections u and v. This was first noted by Batdorf,[9] who showed that the boundary conditions correspond to

$$w = w_{,xx} = v = 0 \text{ along } x = \text{const} \quad (8)$$

$$w = w_{,yy} = u = 0 \text{ along } y = \text{const} \quad (9)$$

Likewise, from Eqs. (2) and (3), it is apparent that consistent thickness-shear force distributions Q_x and Q_y must be of the form

$$Q_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} b_{mn}\cos(m\pi x/a)\sin(n\pi y/b) \quad (10)$$

$$Q_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn}\sin(m\pi x/a)\cos(n\pi y/b) \quad (11)$$

The linear differential operators H_1, H_2, and H_3 are defined such that the functions $H_1(w,Q_x,Q_y)$, $H_2(w,Q_x,Q_y)$, and $H_3(w,Q_x,Q_y)$ are identically equal to the left-hand sides of Eqs. (1), (2), and (3), respectively. Then the Galerkin formulation, in an abbreviated form, may be represented by

$$\int_0^b\int_0^a H_1(w,Q_x,Q_y)(\partial w/\partial a_{mn})dxdy = 0 \quad (12)$$

$$\int_0^b\int_0^a H_2(w,Q_x,Q_y)(\partial Q_x/\partial b_{mn})dxdy = 0 \quad (13)$$

$$\int_0^b\int_0^a H_3(w,Q_x,Q_y)(\partial Q_y/\partial c_{mn})dxdy = 0 \quad (14)$$

where a_{mn}, b_{mn}, and c_{mn} are the Fourier coefficients of the assumed modal functions w, Q_x, and Q_y, respectively.

Completing the substitutions and integrations over the length and width of the plate, one obtains the simultaneous equations for the buckling of the plate. These equations comprise a doubly infinite set of equations in terms of the axial and circumferential wave numbers m and n. These equations are

$$(A_{mn}^1 + A_{mn}^2)a_{mn} + (16/\pi^2)\sum_i\sum_j A_{ij}^3 a_{ij}\delta_{mnij} + A_{mn}^4 b_{mn} + A_{mn}^5 c_{mn} = 0 \quad (15)$$

$$A_{mn}^6 a_{mn} + A_{mn}^7 b_{mn} + A_{mn}^8 c_{mn} = 0 \quad (16)$$

$$A_{mn}^9 a_{mn} + A_{mn}^{10} b_{mn} + A_{mn}^{11} c_{mn} = 0 \quad (17)$$

where the coefficients A_{mn}^i are defined in Appendix B.

These equations are then combined into a single equation involving only the normal deflection coefficient a_{mn} as shown below:

$$a_{mn}\left[(A_{mn}^1 + A_{mn}^2) - \frac{A_{mn}^5 A_{mn}^6}{A_{mn}^8} - \left(A_{mn}^4 - \frac{A_{mn}^5 A_{mn}^7}{A_{mn}^8}\right)\frac{A_{mn}^9 - (A_{mn}^6 A_{mn}^{11}/A_{mn}^8)}{A_{mn}^{10} - (A_{mn}^7 A_{mn}^{11}/A_{mn}^8)}\right] + (32N_{xy}/ab)\sum_i\sum_j a_{ij}\delta_{mnij} = 0 \quad (18)$$

Equation (18) is now in the familiar form of an eigenvalue problem. Writing the equation for as many wave numbers as required for convergence and separating even and odd modes, one obtains directly values of the shear load N_{xy}, the minimum of which is the buckling load $(N_{xy})_{cr}$.

The numerical solution of Eq. (18) results in a matrix formulation of the eigenvalue problem for the even and odd modes with the size of the matrix dependent upon the number of terms required for convergence:

$$\lambda = (ab)/(32N_{xy}) \quad (19)$$

$$\lambda[P_{\text{even}}]\{a_{ij}\} = [T_{\text{even}}]\{a_{ij}\} \qquad (i+j \text{ even}) \quad (20)$$

$$\lambda[P_{\text{odd}}]\{a_{ij}\} = [T_{\text{odd}}]\{a_{ij}\} \qquad (i+j \text{ odd}) \quad (21)$$

where $[P]$ and $[T]$ are matrices deduced readily from Eq. (18).

In general the resulting formulation of the solution of the eigenvalue problem requires the solution of a nonsymmetric matrix. This can either be solved directly by a special numerical routine designed for such a formulation or made symmetric by a transformation technique[10] and then solved.

Numerical Results

As a check for the formulation, five sample cases were analyzed using a computer routine written for the IBM 1130 computer. The five cases were: I) a curved sandwich panel with orthotropic (glass-fabric reinforced plastic) facings and orthotropic (aluminum hexagonal-cell honeycomb) core[2,3]; II) a flat sandwich panel with isotropic (stainless steel) facings and core[11]; III) a thin, homogeneous, isotropic panel (aluminum);[8] IV) a thin, fiber-reinforced, orthotropic panel (boron-epoxy);[12] V) same as Case IV, except here thickness-shear flexibility is included.

For Case I, the present results were compared with test data reported in Ref. 3 and as well as very approximate results obtained in Ref. 3 by adding the buckling load of a flat panel and a curved panel, as suggested by Kuenzi.[13] The appropriate material properties, geometric data, and results are presented in Table 1. It is seen that the present results are somewhat lower than the Ref. 3 approximate result, but considerably higher than the experimentally determined value. This lack of agreement is believed to be, at least partially, due to experimental difficulties in actually achieving the boundary conditions assumed in the analysis. This is borne out by the fact that the buckles were somewhat localized at the corners. However, it is encouraging to note that the present prediction is somewhat lower than the simplified prediction recommended in Ref. 14, and thus closer to the experimental value than the latter.

In Case II, the present result compared very favorably (within 2.4%) with the analytical result reported in Ref. 11, as can be seen in Table 1. The result for Case III agreed very well (within 0.13%) with the value reported by Batdorf.[8]

For Case IV, the dimensionless properties and geometric data used by Ashton and Whitney[12] were specialized to those given in Table 1. These properties are approximately equal to those of thin, unidirectional, boron-epoxy composite material, except that the in-plane shear modulus is excessively large. As can be seen in Table 1, the present result, using five

Table 1 Numerical Results

Panel Characteristics	Case I	Case II	Case III	Case IV	Case V
		Material Properties			
E_x,10^6psi	4.19	30.0	10.5	30.0	30.0
E_y,10^6psi	4.01	30.0	10.5	3.00	3.00
G_{xy},10^6psi	0.562	11.5	4.0	2.07	2.07
ν_{xy}	0.15	0.30	0.30	0.30	0.30
ν_{yx}	0.15	0.30	0.30	0.030	0.030
G_{xz},10^6psi	0.032	0.029	∞	∞	2.07
G_{yz},10^6psi	0.0183	0.029	∞	∞	1.16
		Geometrical Parameters			
a,in.	49.0	40.0	9.215	20.0	20.0
b,in.	49.0	20.0	6.143	20.0	20.0
t,in.	0.021	0.008	0.060	0.100	0.100
c,in.	0.30	0.25	N.A.[a]	N.A.	N.A.
R,in.	21.94	∞	20.0	∞	∞
		Buckling Loads $(N_{xy})_{cr}$,lb/in			
Present work	630	1278	778	194.9	194.6
Others	227 exp. 685 calc.	1248 calc.	777 calc.	194.9 calc.	—

[a] N.A. denotes not applicable.

terms in the Galerkin solution coincided with the Ref. 12 result, which was obtained by a five-term Rayleigh-Ritz analysis. This is considered to be a very stringent check of the validity of the present analysis.

Case V was added to illustrate the effect of thickness-shear flexibility on the shear buckling load of a composite-material panel. All of the dimensions and properties are the same as for Case IV, except that the thickness-shear stiffness are not assumed to increase without bound. The longitudinal thickness-shear modulus was set equal to the in-plane shear modulus, since the material is assumed to be unidirectional and to be transversely isotropic with its plane of isotropy being the plane normal to the fibers. The transverse thickness-shear modulus was chosen to be such that the G_{yz}/G_{xz} ratio coincides with the experimental results reported in Ref. 14 for boron-epoxy composite. As can be seen in Table 1, the effect of thickness-shear flexibility, in going from Case IV to Case V, is negligible. Thus, in composite-material plates, the thickness-shear effect can be neglected except when the thickness is great.

Design Curves

In the general case, the buckling load depends upon twelve material and geometrical parameters: $a,b,D_x,D_y,D_{xy},D_{qx},D_{qy}$, R,S_x,S_y,S_{xy}, and ν_{xy}. Thus, it is impractical to present results of a general nature. However, for the case of a flat panel ($R^{-1}=0$) without shear flexibility ($D_{qx}^{-1}=D_{qy}^{-1}=0$), these quantities, as well as the terms containing S_x,S_y, and S_{xy}, vanish.

Table 2 Material data used in generating the design curves

Curve	Material	Ref.	D_x/D_y	$\nu_{xy}+(D_{xy}/D_y)$
1	Isotropic	—	1.000	1.000
2	Longitudinal glass cloth-epoxy	2,3	1.044	0.424
3	Longitudinal boron-epoxy	16	9.043	0.950
4	Transverse boron-epoxy	16	0.110	0.099
5	Longitudinal graphite-epoxy	17	25.0	1.248
6	Transverse graphite-epoxy	17	0.040	0.0499

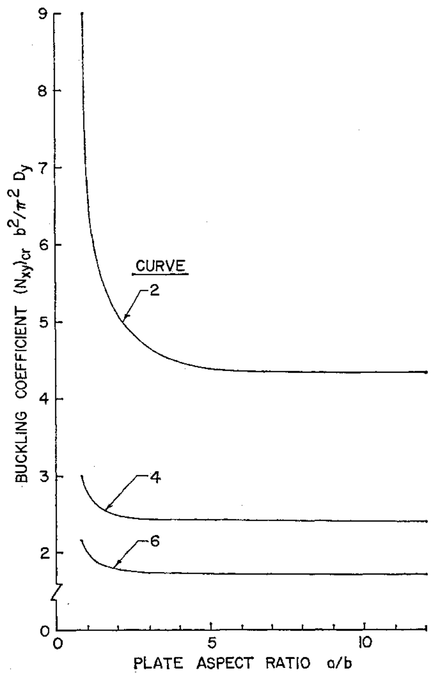

Fig. 1 Buckling curves 1,3,5 (material data from Table 2).

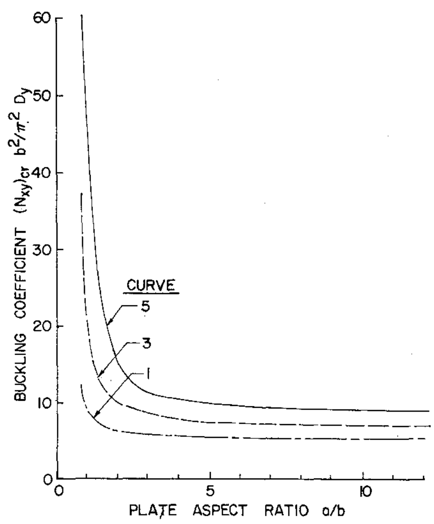

Fig. 2 Buckling curves 2,4,6 (material data from Table 2).

Then the buckling load can be expressed in the following dimensionless form

$$(N_{xy})_{cr}b^2/\pi^2 D_y$$

as a function of the dimensionless geometrical parameter a/b and the two dimensionless material parameters D_x/D_y and $\nu_{xy}+(D_{xy}/D_y)$.

Using the material parameters for various composite materials as listed in Table 2, the design curves shown in Figs. 1 and 2 were calculated.

Conclusion

An analysis has been carried out for the elastic stability of orthotropic curved panels loaded by edgewise shear, taking into account thickness-shear flexibility. A numerical result obtained by this technique was compared with the only test result available for the title problem and predicted a higher buckling load but lower than the result of a simplified analysis. Also comparisons were made with previous analytical results with excellent agreement for various cases of flat and curved panels of sandwich and nonsandwich configurations and of isotropic and composite materials.

Appendix A Stiffness Formulas

Case I Sandwich with Orthotropic Facings, Orthotropic Core

$$S_x = 2tE_x \quad \text{(A1)}$$

$$S_y = 2tE_y \quad \text{(A2)}$$

$$S_{xy} = 2tG_{xy} \quad \text{(A3)}$$

$$D_x = (S_x/4)(c+t)^2(1-\nu_{xy}\nu_{yx})^{-1} \quad \text{(A4)}$$

$$D_y = (S_y/4)(c+t)^2(1-\nu_{xy}\nu_{yx})^{-1} \quad \text{(A5)}$$

$$D_{xy} = (S_{xy}/2)(c+t)^2 \quad \text{(A6)‡}$$

$$D_{qx} = G_x t(c+t)^2/c \quad \text{(A7)}$$

$$D_{qy} = G_y t(c+t)^2/c \quad \text{(A8)}$$

Case II Sandwich with Isotropic Facings, Isotropic Core

$$\nu_{xy} = \nu_{yx} = \nu_s \quad \text{(A9)}$$

$$S_x = S_y = 2E_s t \quad \text{(A10)}$$

$$S_{xy} = E_s t/(1+\nu_s) \quad \text{(A11)}$$

$$D_x = D_y = (E_s t/2)(c+t)^2/(1-\nu_s^2) \quad \text{(A12)}$$

$$D_{xy} = D_x(1-\nu_s) \quad \text{(A13)}$$

$$D_{qx} = D_{qy} = G_z(c+t)^2/c \quad \text{(A14)}$$

Case III Thin, Homogeneous, Isotropic Panel

$$\nu_{xy} = \nu_{yx} = \nu \quad \text{(A15)}$$

$$S_x = S_y = Et_0 \quad \text{(A16)}$$

$$S_{xy} = \tfrac{1}{2}Et_0/(1+\nu) \quad \text{(A17)}$$

$$D_x = D_y = (Et_0^3/12)(1-\nu^2)^{-1} \quad \text{(A18)}$$

$$D_{xy} = D_x(1-\nu) \quad \text{(A19)}$$

$$D_{qx} = D_{qy} \to \infty \quad \text{(A20)}$$

Case IV Thin, Orthotropic Panel

$$S_x = t_0 E_x \quad \text{(A21)}$$

$$S_y = t_0 E_y \quad \text{(A22)}$$

$$S_{xy} = t_0 G_{xy} \quad \text{(A23)}$$

$$D_x = (E_x t_0^3/12)(1-\nu_{xy}\nu_{yx})^{-1} \quad \text{(A24)}$$

$$D_y = (E_y t_0^3/12)(1-\nu_{xy}\nu_{yx})^{-1} \quad \text{(A25)}$$

$$D_{xy} = G_{xy}t_0^3/6 \quad \text{(A26)}$$

$$D_{qx} = D_{qy} \to \infty \quad \text{(A27)}$$

Case V Orthotropic Panel Including Thickness-Shear Flexibility

$S_x, S_y, S_{xy}, D_x, D_y$ and D_{xy} are the same as in Case IV, Eqs. (A21–A27).

$$D_{qx} = (5/6)t_0 G_{xz} \quad \text{(A28)}$$

$$D_{qy} = (5/6)t_0 G_{yz} \quad \text{(A29)}$$

‡ Here the twisting stiffness D_{xy} is defined as is conventional in sandwich plate and shell theory[1–3,6–7,15]. In thin plate and shell theory, including composites,[12] it is customary to define D_{xy} to be one-half of the present definition so that it appears in such equations multiplied by a factor of 2.

Appendix B Definitions of Coefficients Appearing in Eqs. (18–20)

$$A_{mn}{}^1 = \pi^4[D_x(m/a)^4 + 2(\nu_{xy}D_y + D_{xy})(mn/ab)^2 + D_y(n/b)^4] \quad \text{(B1)}$$

$$A_{mn}{}^2 = R^{-2}(m/a)^4[S_y{}^{-1}(m/a)^4 + (S_{xy}{}^{-1} - 2\nu_{xy}S_x{}^{-1})(mn/ab)^2 + S_x{}^{-1}(n/b)^4]^{-1} \quad \text{(B2)}$$

$$A_{mn}{}^3 = 2\pi^2 N_{xy}/ab \quad \text{(B3)}$$

$$A_{mn}{}^4 = (\pi^3/D_{qx})[D_x(m/a)^3 + (\nu_{xy}D_y + D_{xy})(m/a)(n/b)^2] \quad \text{(B4)}$$

$$A_{mn}{}^5 = (\pi^3/D_{qy})[D_y(n/b)^3 + (\nu_{yx}D_x + D_{xy})(m/a)^2(n/b)] \quad \text{(B5)}$$

$$A_{mn}{}^6 = \pi^3[D_x(m/a)^3 + (\nu_{yx}D_y + D_{xy})(m/a)(n/b)^2] \quad \text{(B6)}$$

$$A_{mn}{}^7 = 1 + (D_x/D_{qx})(\pi m/a)^2 + \tfrac{1}{2}(D_{xy}/D_{qx})(\pi n/b)^2 \quad \text{(B7)}$$

$$A_{mn}{}^8 = (\pi^2/D_{qy})(mn/ab)(\nu_{yx}D_x + \tfrac{1}{2}D_{xy}) \quad \text{(B8)}$$

$$A_{mn}{}^9 = \pi^3(n/b)[D_y(n/b)^2 + (\nu_{xy}D_y + D_{xy})(m/a)^2] \quad \text{(B9)}$$

$$A_{mn}{}^{10} = (\pi^2/D_{qx})(mn/ab)(\nu_{xy}D_y + \tfrac{1}{2}D_{xy}) \quad \text{(B10)}$$

$$A_{mn}{}^{11} = 1 + (D_y/D_{qy})(\pi n/b)^2 + \tfrac{1}{2}(D_{xy}/D_{qy})(\pi m/a)^2 \quad \text{(B11)}$$

$$\delta_{mnij} = \begin{cases} mnij/(m^2 - i^2)(n^2 - j^2), \text{ when } (m \pm i) \text{ and } (n \pm j) \text{ are odd integers} \\ 0, \text{ otherwise} \end{cases} \quad \text{(B12)}$$

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